Teacher Notes – KEY

CW#67H: Reference Angles
Honors Geometry

CRS	FUN 703 - Exhibit knowledge of unit circle trigonometry.
Objective	10.14 – Find reference angles 10.15 – Use reference angles to evaluate functions

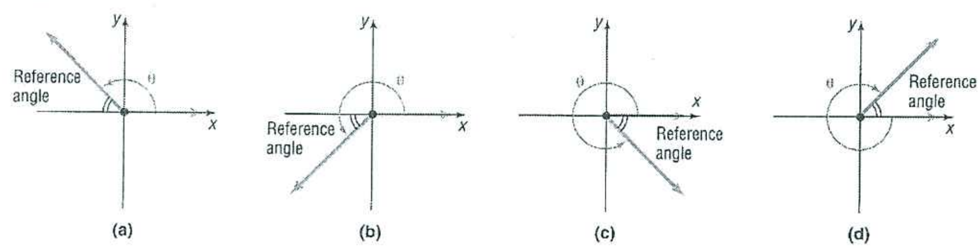
Review:

NAME	ABBREVIATION	RATIO	NAME	ABBREVIATION	RATIO
Sine	sin (y-value)	opp/hyp	cosecant	csc	hyp/opp
Cosine	cos (x-value)	adj/hyp	secant	sec	hyp/adj
Tangent	$\frac{\sin}{\cos}$ tan (y/x)	opp/adj	cotangent	cot	adj/opp

What if $\theta = 150^o$, can we still use the unit circle? csc = 1/sine sec = 1/cosine
Quotient Id. cot = 1/tangent

Def >> Let θ denote an angle that lies in a quadrant. The acute angle formed by the terminal side of θ and the x-axis is called the **reference angle** of θ. > 0, 180, 360

Figure 57



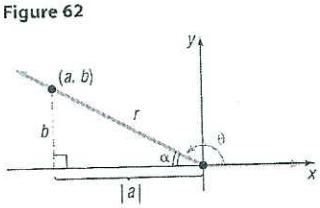
Although formulas can be given for calculating reference angles, usually it is easier to find the reference angle for a given angle by making a quick sketch of the angle.

Find the Reference Angles for:

a) 150° → 30°
b) -45°→45°
c) 9π/4 → π/4
d) -5π/6 → π/6

The advantage of using reference angles is that, except for the correct sign, the values of the trigonometric functions of a general angle θ equal the values of the trigonometric functions of its reference angle.

Figure 62



Find the exact value of each of the following trigonometric functions using reference angles:

a) sin 135°
= sqrt(2)/2
ref = 45

b) cos 600°
= -1/2
ref = -60

c) cos 17π/6
= - sqrt(3)/2
ref = -π/6

d) tan (-π/3)
= - sqrt(3)
ref = - π/3

THE PROCESS:

-if the angle θ is a quadrant angle, draw the angle and pick a point on its terminal side, apply the definitions of the trig functions as usual.

-if the angle θ lies in a quadrant:

1. Find the reference angle α of θ
2. Find the value of the trig functions at α
3. Adjust the sign (+ or -) of the value of the trig function based on the quadrant in which θ lies.

QI (+,+) sinθ cosθ QII (-,+) tanθ − cosθ sinθ QIII (-,-) tanθ + cosθ sinθ QIV (+,−) tan θ − cosθ sinθ

Name: *Key!* TP: _______

HW#67H: Reference Angles
Due Monday, Feb. 25th
Honors Geometry

Failure to show all work and write in complete sentences will result in LaSalle!

1) Evaluate, if possible, the **secant** function and **cotangent** function of the 4 quadrantal angles:

secant = hyp/adj cot = adj/opp

a. $\theta = 0° = 0$ (1,0)

$\sec\theta = 0/1 = 0$

$\cot\theta = 1/0 =$ undefined

b. $\theta = 90° = \frac{\pi}{2}$ (0,1)

$\sec\theta = 1/0 =$ undefined

$\cot\theta = 0/1 = 0$

c. $\theta = 180° = \pi$ (−1,0)

$\sec\theta = 0/-1 = 0$

$\cot\theta = -1/0 =$ undefined

d. $\theta = 270° = \frac{3\pi}{2}$ (0,−1)

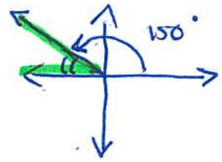
$\sec\theta = -1/0 =$ undefined

$\cot\theta = 0/-1 = 0$

2) Name the reference angle for the given angles:

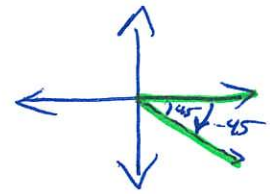
a. 150^0

30°



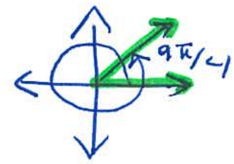
b. -45^0

45°



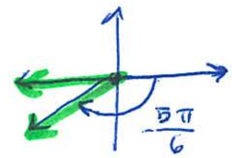
c. $\frac{9\pi}{4}$

$\pi/4$



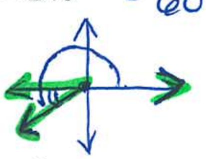
d. $-\frac{5\pi}{6}$

$\pi/6$

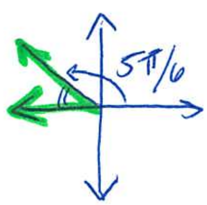


3) Find the reference angle θ', for each of the following angles:

a. $\theta = 240°$ = 60°

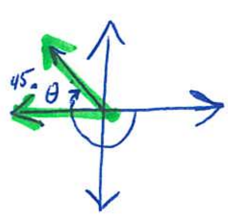


b. $\theta = \frac{5\pi}{6}$ = $\pi/6$



c. $\theta = -\frac{5\pi}{4}$

$\frac{\pi}{4}$



4) Use reference angles to find the exact value of the following trig functions:

a. $\cos\theta = 225°$ Q III

$\cos\theta = \frac{-\sqrt{2}}{2}$

Ref angle = 45°

$\cos\theta$ is negative (x)

b. sin ~~$\csc\theta =$~~ $\frac{7\pi}{6}$ Q III

sin = y-value

Ref angle = 30°

$\sin\theta = -1/2$

c. ~~$\cot\theta = 225$~~ tan 150°

y/x $\frac{1/2}{-\sqrt{3}/2}$

Ref ∠ = 30°

Q II

$\frac{1}{2} * \frac{2}{-\sqrt{3}} = \frac{2}{-2\sqrt{3}} = \frac{2\sqrt{3}}{-6}$

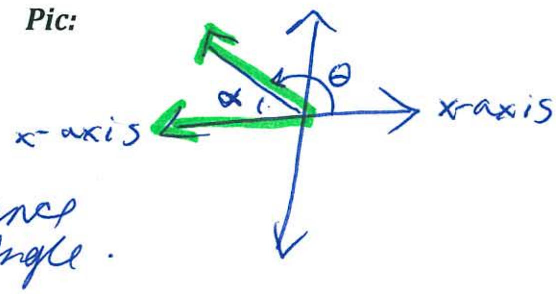
$= -\sqrt{3}/3$

5) Explain why $\tan\theta = \frac{\pi}{2}$ is undefined.

$\pi/2$ is $90°$ angle with (x, y) of $(0, 1)$. The ratio of tan at $\pi/2$ is y/x or specifically $1/0$. Dividing by zero is undefined thus Tan θ $\pi/2$ is undefined.

6) What is a reference angle? How can reference angles be used to evaluate trig functions? Give an example with your description.

Pic:



Explanation:

Refrence angles mark the acute angle formed by the terminal side of θ and x-axis. Always acute!

Determine whether each statement makes sense or does not make sense, and explain your reasoning.

7) This angle θ is in a quadrant for which $\sin\theta < 0$ and $\csc\theta > 0$ (Think about the value of x and y in each quadrant)

$\sin\theta < 0$ where y is negative and $\csc\theta > 0$ or $1/\sin\theta$ is positive. This is imposisible! $\sin\theta$ can't be positive and negative :(

8) I am given the following: $\tan\theta = \frac{3}{5}$ therefore I can conclude that y = 3 and x = 5.

Name: Key & Notes TP: ________

CW#68H: Reference Angles (2)
Honors Geometry

CRS	FUN 703 - Exhibit knowledge of unit circle trigonometry.
Objective	10.14 – Find reference angles(2) 10.15A – Determine the quadrant [including naming quadrants as an interval] of an angle given the sign(s) of trig function(s) (4)

All things important! Intro the SWAG sheet. ≫ stuff we all get but need a reminder every once in a while :)

Solutions will be posted on wednes.

"Swag is earned not given"

Quotient Identities:

$$\tan\,\theta = \frac{\sin\theta}{\cos\theta} \qquad \cot\,\theta = \frac{\cos\theta}{\sin\theta}$$

Pythagorean Identities: $\sin^2\theta + \cos^2\theta = 1$

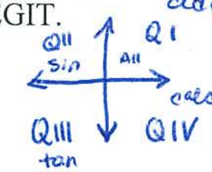
Reciprocal Identities: $\csc\theta = \frac{1}{\sin\theta}$ $\qquad \sec\theta = \frac{1}{\cos\theta}$ $\qquad \cot\theta = \frac{1}{\tan\theta}$

Review: of Friday

If $90° < \theta < 180°$, then we know θ lies in which quadrant? And the reference angle $\theta' = 180° - \theta$ QII	**Name the Quadrants for the following reference angles, given the inequalities. Remember to label the reference angle θ'.**
If $180° < \theta < 270°$, then we know θ lies in which quadrant? And the reference angle $\theta' = \theta - 180°$ QIII	If $270° < \theta < 360°$, then we know θ lies in which quadrant? And the reference angle $\theta' = 360° - \theta$ QIV

MUST WATCH THE SIGNS! If it lands in quadrant III what are the positive trig functions in that quad? LOOK @ 4b on CW. there is a property with trig functions - $\sin(\frac{2\pi}{3})$ odd -

Ok, I have a trick for the sign values so you NEVER well almost never have to look at your unit circle again to learn it. All Students Take Calc. (**QI**: All positive **QII**: Sin/Csc **Q III**: Tan/Cot **QIV**: Cos/Sec) LEGIT.



What about the quadrant angles?

θ **(Degrees)**	θ **(Radians)**	**sin** θ	**cos** θ	**tan** θ	**csc** θ	**sec** θ	**cot** θ
0	0	0	1	0	Not Defined	1	undefined
90°	$\frac{\pi}{2}$	1	0	not defined	1	undef.	0
180°	π	0	-1	0	Not def.	-1	undef
270	$\frac{3\pi}{2}$	-1	0	not defined	-1	undef.	0
360	2π	0	1	Not Defined 0	Not defi	1	0 undef.

PUSH IT TO THE LIMIT.

CLASSROOM COPY – DO NOT WRITE ON THIS!!!!!

CW#68H: Reference Angles (2)
Honors Geometry

CRS	FUN 703 - Exhibit knowledge of unit circle trigonometry.
Objective	10.14 – Find reference angles(2) 10.15A – Determine the quadrant [including naming quadrants as an interval] of an angle given the sign(s) of trig function(s) (4)

Practice Problems: Mixed Review

1) Find the exact value of each expression. Do **not** use a calculator.

a. $4(\cos 45^0) - 2(\sin 45^0)$

$4(\sqrt{2}/2) - 2(\sqrt{2}/2)$

$2\sqrt{2} - \sqrt{2} = \boxed{\sqrt{2}}$

b. $2(\sin 45^0) + 4(\cos 30^0)$

$2(\sqrt{2}/2) + 4(\sqrt{3}/2)$

$\sqrt{2} + 2\sqrt{3} = \boxed{2\sqrt{3} + \sqrt{2}}$

c. $6(\tan 45^0) - 8(\cos 60^0)$

$6(1) - 8(1/2)$

$6 - 4 = \boxed{2}$

d. $4 + \tan^2\frac{\pi}{3}$

$4 + (\sqrt{3})^2 = 4 + 3 = \boxed{7}$

2) Find the exact value of each expression. Do **not** use a calculator.

a. $\sec\frac{\pi}{4} + 2\csc\frac{\pi}{3}$

$\sqrt{2} + 2\left(\frac{2\sqrt{3}}{3}\right) = \boxed{\sqrt{2} + \frac{4\sqrt{3}}{3}}$

b. $\tan\frac{\pi}{4} + \cot\frac{\pi}{4}$

$1 + 1 = \boxed{2}$ ☺

c. $\sec^2\frac{\pi}{6} - 4$

$\left(\frac{2\sqrt{3}}{3}\right)^2 - 4 = \frac{12}{9} - 4 = \boxed{-8/3}$

tan θ 2

d. $1 + \tan^2 30^0 - \sec^2 45^0$

$(\tan 30)^2$

$1 + (\sqrt{3}/3)^2 - (\sqrt{2})^2 = 1 - \frac{3}{9} - 2 = \boxed{-4/3}$

3) Find the reference angle for θ, **and** list the **positive** trig functions for θ based on its graph.

a. $\theta = 210°$

QIII tan/cot (+)

Ref = 30°

b. $\theta = 440°$

QI all positive

Ref = 60°

c. $\theta = \frac{15\pi}{4}$ = 675° QIV

Ref = 45° cos/sec (+)

4) Use reference angles to find the **exact** value of the following trig functions: **WATCH YOUR SIGNS!**

a. $\tan\frac{8\pi}{3}$ QII Find ref ∠

$\tan\frac{\pi}{\cancel{6}} = \tan 8\pi/3 = -\sqrt{3}$

b. $\sin\left(-\frac{2\pi}{3}\right)$ QIII

$-\sin\left(\frac{2\pi}{3}\right)$

$-\sin(\pi/3) = -\sqrt{3}/2$

c. $\cos(-2\pi)$

$-\cos(2\pi) = \boxed{-1}$

PUSH IT TO THE LIMIT.

Name: ______________________________ TP: ________

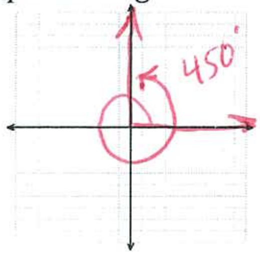
HW#67H: Reference Angles (2)
Due Tuesday, Feb. 26th
Honors Geometry

Failure to show all work will result in LaSalle. **REMEMBER TO WATCH YOUR SIGNS!**

1) Find the **exact** value of the given function:

csc 450^0

a. Graph the angle:



b. What is the reference angle?

90°

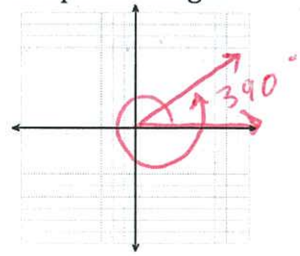
c. What is the value of **csc 450^0** ?

1 (using my SWAG sheet)

2) Find the **exact** value of the given function:

cot 390^0

a. Graph the angle:



b. What is the reference angle?

30°

c. What is the value of **cot 390^0** ?

$\sqrt{3}$ (using my swag sheet)

3) Find the **exact** value of the given function:

$\mathbf{cos}\ \frac{33\pi}{4} = \cos\left(\frac{\pi}{4} + \frac{32\pi}{4}\right)$

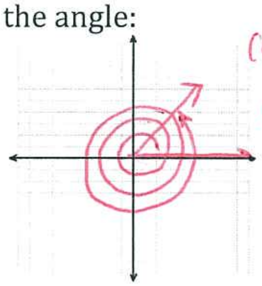
$\cos\left(\frac{\pi}{4} + 8\pi\right)$

$\cos((2\pi)4)$

$\cos(\pi/4)$

4

a. Graph the angle:



4x!

b. What is the reference angle?

$\pi/4$

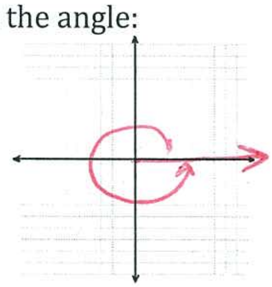
c. What is the value of $\mathbf{cos}\ \frac{33\pi}{4}$?

$\sqrt{2}/2$

4) Find the **exact** value of the given function:

tan 2π

a. Graph the angle:



b. What is the reference angle?

2π

c. What is the value of **tan** 2π ?

0

Teacher Notes – KEY

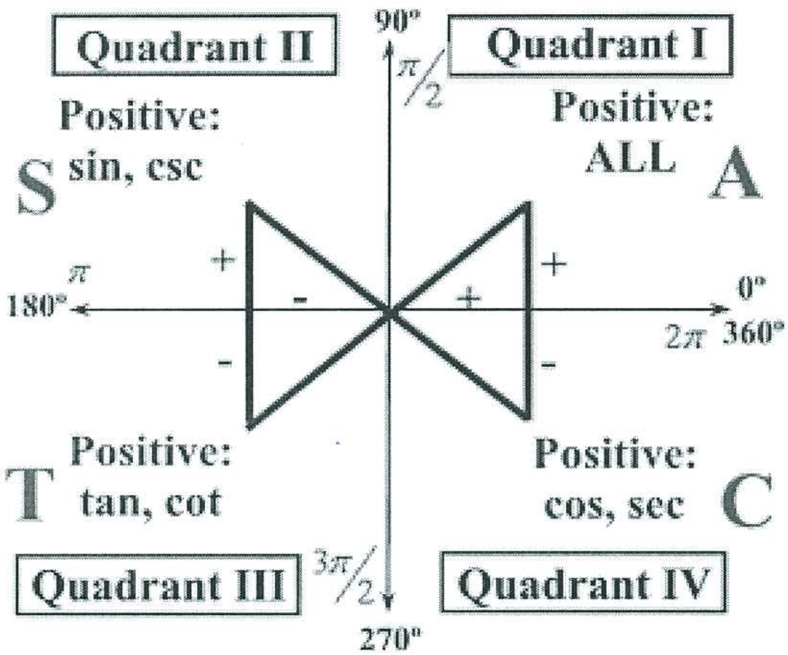
CW#69H: Trig of All Angles
Honors Geometry

CRS	FUN 703 - Exhibit knowledge of unit circle trigonometry.
Objective	10.15B - Evaluate the 6 trigonometric functions of any angle using reference angles.

Review:

If θ is not a quadrantal angle, the <u>sign</u> of a trig function depends on the quadrant in which θ lies:

Draw Picture:



*A way for students to remember is the saying:

"**A** **S**mart **T**rig **C**lass" or "**A**ll **S**tudents **T**ake **C**alculus"

A S T C

*All positive in Q1

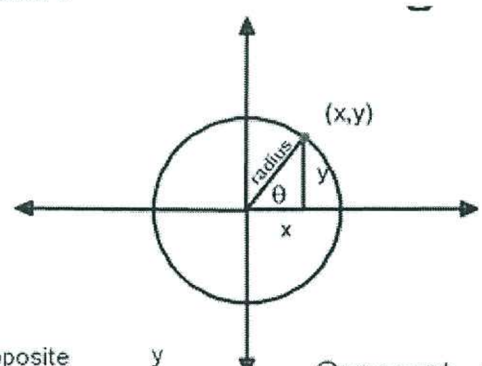
*Sine (and reciprocal function cosecant) positive in Q2

*Tangent (and reciprocal cotangent) positive in Q3

*Cosine (and reciprocal secant) positive in Q4

Example 1: If $\tan\theta < 0$ and $\cos\theta > 0$, name the quadrant in which θ lies. → QIV

Unit circle w/ radius r



Always connect to the x-axis!

Sine $= \dfrac{\text{Opposite}}{\text{Hypotenuse}} = \dfrac{y}{r}$

Cosecant $= \dfrac{\text{Hypotenuse}}{\text{Opposite}} = \dfrac{r}{y}$

Cosine $= \dfrac{\text{Adjacent}}{\text{Hypotenuse}} = \dfrac{x}{r}$

Secant $= \dfrac{\text{Hypotenuse}}{\text{Adjacent}} = \dfrac{r}{x}$

Tangent $= \dfrac{\text{Opposite}}{\text{Adjacent}} = \dfrac{y}{x}$

Cotangent $= \dfrac{\text{Adjacent}}{\text{Opposite}} = \dfrac{x}{y}$

*y will always be the side opposite
*x will always be the side adjacent
*the hypotenuse is the radius, r

Ex 2. Let $(\overset{x}{8}, \overset{y}{-10})$ be a point on the terminal side of an angle θ in standard position. Find the exact value of each of the six trig functions of θ.

Classroom Copy –*DO NOT WRITE ON!*

CW#69H: Trig of All Angles
Honors Geometry

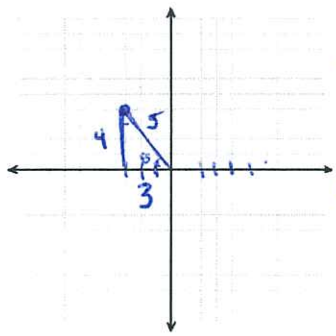
CRS	FUN 703 - Exhibit knowledge of unit circle trigonometry.
Objective	10.15B - Evaluate the 6 trigonometric functions of any angle using reference angles.

WATCH YOUR SIGNS!!

Directions: A point on the terminal side of an angle θ is given. Find the exact value of **sec**, **csc**, and **cot**.

1. (-3,4)

$9+16=25$



$\csc\theta = \frac{5}{4}$

$\sec\theta = -\frac{5}{3}$

$\cot = -\frac{3}{4}$

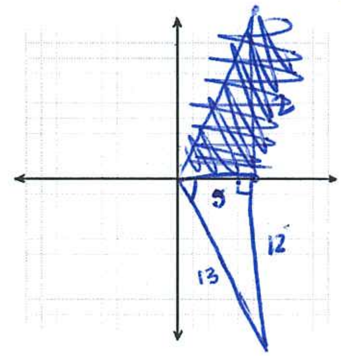
Check signs! Quad II only sin & csc (+)

2. (5,-12)

$\csc = -13/12$

$\sec = 13/5$

$\cot = -5/12$



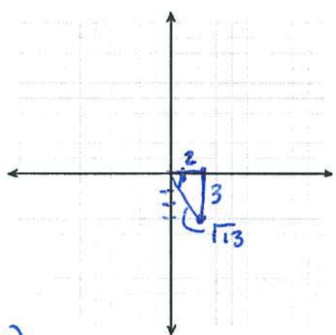
$25+144$

169

QIV cos & sec(+)

3. (2, -3)

$4+9=13$



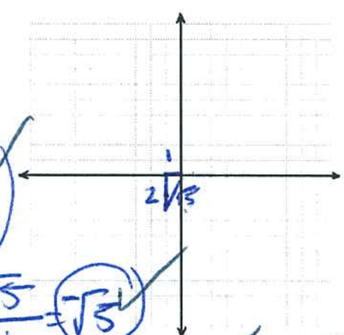
$\csc\theta = \frac{-\sqrt{13}}{3}$

$\sec\theta = \frac{\sqrt{13}}{2}$

$\cot\theta = -2/3$

QIV sec(+)

4. (-1, -2)



$\csc\theta = \frac{-\sqrt{5}}{2}$

$\sec\theta = \frac{\sqrt{5}}{1} = -\sqrt{5}$

$\cot\theta = 2/1 = 2$

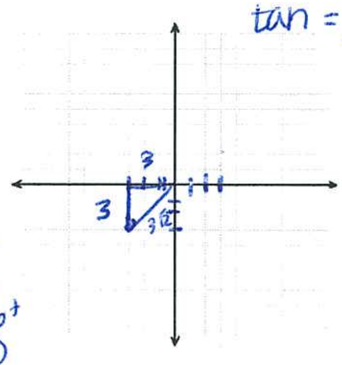
tan & cot QIII (+)

5. (-3,-3)

$\csc = 3\sqrt{2}/3 = -\sqrt{2}$

$\sec = 3\sqrt{2}/3 = -\sqrt{2}$

$\tan = \frac{3}{3} = 1$



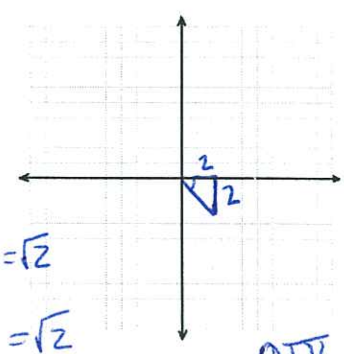
QIII tan, cot (+)

$3^2+3^2 = 18$

$hyp = \sqrt{18} = 3\sqrt{2}$

45, 45, 90

6. (2, -2)



$hyp = 2\sqrt{2}$

$\csc\theta = \frac{2\sqrt{2}}{2} = \sqrt{2}$

$\sec\theta = \frac{2\sqrt{2}}{2} = \sqrt{2}$

$\tan\theta = \frac{2}{2} = -1$

QIV cos(+) sec(+)

Directions: Use reference angles to find the exact value of each expression. **WATCH YOUR SIGNS!**

7. $\sin 510^{\circ}$

$= 1/2$

8. $\cos 600^{\circ}$

$= -1/2$

9. $\cot 330^{\circ}$

$\sqrt{3}/2$

10. $\sin \frac{3\pi}{4}$ $= \frac{1}{\sqrt{2}} = \sqrt{2}/2$

11. $\tan (7\pi)$

$= 0$

12. $\csc 300^{\circ}$

$\csc(60) = -\frac{2\sqrt{3}}{3}$

13. **Find the exact value: do NOT use a calculator.**

$\cos 540^{\circ} - \tan(-405^{\circ})$

$(-1) - (-1) = 0$

14. Find the **exact** value:

$\sin 270^{\circ} + \cos(-180^{\circ})$

$(-1) + (-1) = -2$

15. Find the **exact** value:

$\tan \pi + \sin \pi$

$0 + 0 = 0$

16. Find the **exact** value:

$6\cos\left(\frac{3\pi}{4}\right) + 2\tan\left(-\frac{\pi}{3}\right)$

$-3\sqrt{2} + (-2\sqrt{3}) = \boxed{-3\sqrt{2}-2\sqrt{3}}$

17. **Review:** Find the three <u>smallest</u> positive angles coterminal to -800°.

-800
$+720$
$= \boxed{80^{\circ}, 440^{\circ}, 800^{\circ}}$

18. **Review:** Find the three <u>smallest</u> positive angles coterminal to 6413°.

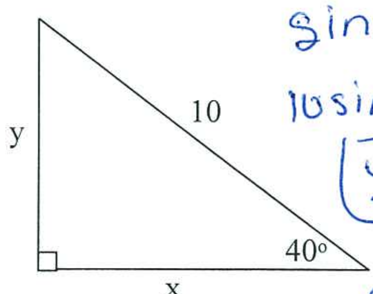
$6413/360 \approx 17.$

$360 \times 17 = 6120$

$6413 - 6120 = 293$

$\boxed{293^{\circ}, 653^{\circ}, 1013^{\circ}}$

19. **Review:** Given the following triangle, find the remaining sides.



$\sin 40 = \frac{y}{10}$

$10\sin 40 = y$

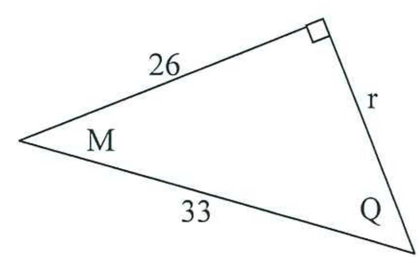
$\boxed{y = 6.428}$

$\cos 40 = \frac{x}{10}$

$10\cos 40 = x$

$\boxed{x = 7.66}$

20. **Review:** Given the following triangle, find the missing angles and side.



$26^2 + r^2 = 33^2$

$676 + r^2 = 1089$

$r^2 = 413$ $r = \sqrt{413}$

Name: Key TP: ________

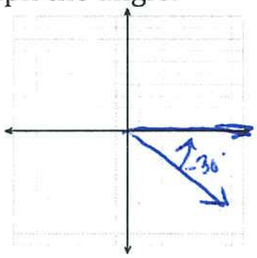
HW#69H: Reference Angles (2)
Due Wednesday, Feb. 27th
Honors Geometry

Failure to show all work will result in LaSalle. **REMEMBER TO WATCH YOUR SIGNS!**

1) Find the **exact** value of the given function:

$$\cot -\frac{\pi}{6} = -30^\circ$$

a. Graph the angle:



b. What is the reference angle?

30°

c. What is the value of **cot** $-\frac{\pi}{6}$?

$-\cot \pi/6 = \boxed{-\sqrt{3}}$

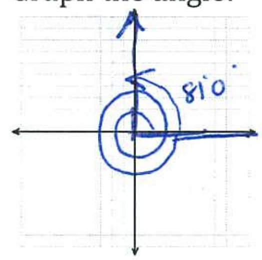
QIV

cos, sec(+)

2) Find the **exact** value of the given function:

$$\csc \frac{9\pi}{2}$$

a. Graph the angle:



b. What is the reference angle?

90°

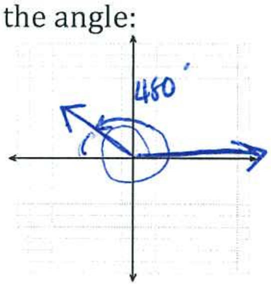
c. What is the value of **csc** $\frac{9\pi}{2}$?

$\csc(90) = 1$

3) Find the **exact** value of the given function:

$$\sin \frac{8\pi}{3}$$

a. Graph the angle:



b. What is the reference angle?

60°

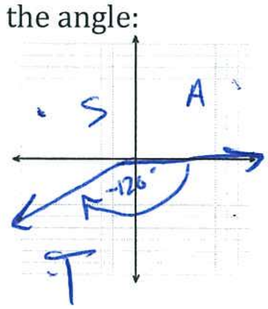
QII c. What is the value of **sin** $\frac{8\pi}{3}$?

$\sqrt{3}/2$

4) Find the **exact** value of the given function:

$$\tan \frac{-2\pi}{3}$$

a. Graph the angle:



b. What is the reference angle?

60°

c. What is the value of **tan** $\frac{-2\pi}{3}$?

$-(\tan 60^\circ) = -\sqrt{3}/1$

$= \sqrt{3}$

Quiz ↳ leader

Mastery Manager

5) Myth or Fact: If cos $\theta > 0$ and cot $\theta < 0$ then θ lies in quadrant IV. Explain your answer.

cos $\theta > 0$, Q III, I and cot $\theta < 0$ quad 1, II, IV, yes it is possible for θ to lie QIV because cos θ is > 0 in QIV and cot $\theta < 0$ in Quad IV. or even QI.

FACT

6) Myth or Fact: If sin $\theta < 0$ and cos $\theta > 0$ then θ lies in quadrant II. Explain your answer.

sin $\theta < 0$ in Q III & Q IV and cos $\theta > 0$ in quad I, IV it would be a myth to say that θ lies in QII.

7) Name the quadrant in which the angle θ lies. If sec $\theta < 0$ and tan $\theta > 0$.

Explain your answer: sec $\theta < 0$ II, III and ta $\theta > 0$ in Q1, QIII thus θ lies in QIII because it's the only quad that defines both values appropriately.

Determine whether each statement makes sense or does not make sense, and explain your reasoning.

8) Tan 225° has a positive solution since its reference angle lies in quadrant II.

Tan > 0 in Q1 & QIII, this statement does not make sense.

9) Csc 300° has a negative solution since its reference angle lies in quadrant IV.

QIV has cos & sec as the only (+) functions in QIV, thus csc 300 must have a negative solution. Makes sense.

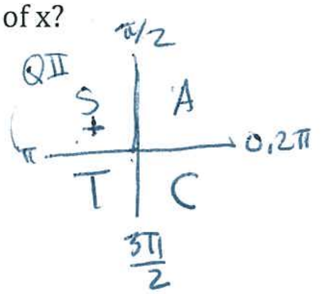
Teacher Notes – KEY

CW#70H: Inverse Trig Functions
Honors Geometry

CRS	FUN 703 - Exhibit knowledge of unit circle trigonometry.
Objective	10.16 – Evaluate inverse trig functions (4)

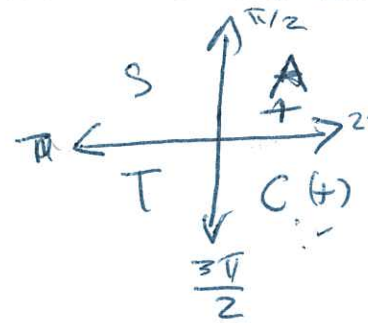
Review:

1) For $0 < x < 2\pi$, if $\sin x > 0$ and $\cos x < 0$, what are the possible values of x?



A. $0 < x < \frac{\pi}{2}$
B. $\frac{\pi}{2} < x < \pi$
C. $\pi < x < \frac{3\pi}{2}$
D. $\frac{3\pi}{2} < x < 2\pi$
E. $0 < x < 2\pi$

2) For $0 < x < 2\pi$, if $\sec x > 0$ and $\tan x < 0$, what are the possible values of x? → cos > 0 too



A. $0 < x < \frac{\pi}{2}$
B. $\frac{\pi}{2} < x < \pi$
C. $\pi < x < \frac{3\pi}{2}$
D. $\frac{3\pi}{2} < x < 2\pi$
E. $0 < x < 2\pi$

So far in this chapter, you have learned to evaluate trigonometric functions of a given angle. In this lesson, you will study the reverse problem—finding an angle that corresponds to a given value of a trigonometric function.

Suppose you were asked to find an angle θ whose sine is 0.5. After considering the problem, you would realize many such angles exist. For instance, the angles

*Have students look at Unit Circle and determine which angles whose sine is ½ → 30, 150,

Since $\frac{\pi}{6}$ and $\frac{5\pi}{6}$ both have a sine value of ½ , we must obtain a unique [only ONE] angle θ such that $\sin\ \theta = ½$.

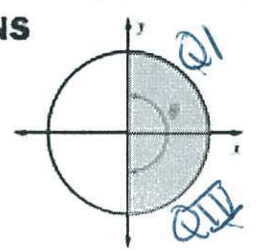
Therefore, we must restrict the domain of the since function. Restricting the domain allows the inverse since, inverse cosine and inverse tangent functions to be defined.

Inverse Trig Functions:

Inverse Sine:

INVERSE TRIGONOMETRIC FUNCTIONS

- **If $-1 \le a \le 1$, then the** inverse sine **of a is an angle θ, written $\theta = \sin^{-1} a$, where $\sin\theta = a$ and $-\frac{\pi}{2} \le \theta \le \frac{\pi}{2}$ (or $-90° \le \theta \le 90°$).**



Why? Look at unit circle:

$\sin\theta = \frac{1}{2}$

$\theta = \sin^{-1}\frac{1}{2}$

$\theta = \frac{\pi}{6}(QI), \frac{5\pi}{6}(QII)$

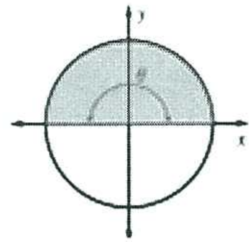
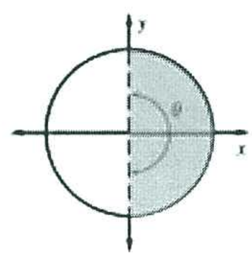
Eliminate QII

$\sin\theta = -\frac{1}{2}$

$\theta = \sin^{-1}(-\frac{1}{2})$

$\theta = \frac{7\pi}{6}(QIII), \frac{11\pi}{6}(QIV)$

Eliminate QIII

Inverse Cosine: • If $-1 \le a \le 1$, then the inverse cosine of a is an angle θ, written $\theta = \cos^{-1} a$, where $\cos\theta = a$ and $0 \le \theta \le \pi$ (or $0° \le \theta \le 180°$). 	Go through the exercise above if needed picking a value for cos
Inverse Tangent: • If a is any real number, then the inverse tangent of a is an angle θ, written $\theta = \tan^{-1} a$, where $\tan\theta = a$ and $-\frac{\pi}{2} < \theta < \frac{\pi}{2}$ (or $-90° < \theta < 90°$). 	Go through the exercise above if needed picking a value for tan

Solve the equation for θ without a calculator. Give your answer in both radians and degrees.

Example 1: $\sin\theta = \frac{\sqrt{3}}{2}$ **Answer:** When $-\frac{\pi}{2} < \theta < \frac{\pi}{2}$, or $-90° < \theta < 90°$, then angle whose sine is $\frac{\sqrt{3}}{2}$ is: $\sin\theta = \frac{\sqrt{3}}{2}$ $\theta = \sin^{-1}(\frac{\sqrt{3}}{2})$ $\theta = \frac{\pi}{3}, 60°$	**Example 2:** $\cos\theta = (-0.5)$ **Answer:** When $0 < \theta < \pi$, or $0° < \theta < 180°$, then angle whose cosine is -0.5 is: $\cos\theta = (-0.5)$ $\theta = \cos^{-1}(-0.5)$ $\theta = \frac{2\pi}{3}, 120°$	**Example 3:** $\tan\theta = (-1)$ **Answer:** When $-\frac{\pi}{2} < \theta < \frac{\pi}{2}$, or $-90° < \theta < 90°$, then angle whose tangent is -1 is: $\tan\theta = (-1)$ $\theta = \tan^{-1}(-1)$ $\theta = -\frac{\pi}{4}, -45°$

Classroom Copy – ***DO NOT WRITE ON!***

CW#70H: Inverse Trig Functions
Honors Geometry

CRS	FUN 703 - Exhibit knowledge of unit circle trigonometry.
Objective	10.16 – Evaluate inverse trig functions (4)

Solve the equation for θ without a calculator. Give your answer in both radians and degrees.

Ex 1: $\sin\theta = \frac{\sqrt{3}}{2}$ QI + QII y value $= 120^\circ$ or $\frac{2\pi}{3}$	**Ex 2:** $\cos\theta = (-0.5)$ = x value	**Ex 3:** $\tan\theta = (-1)$ = y/x

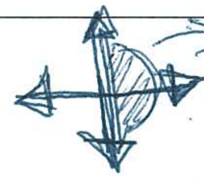
Practice Problems: Use a unit circle to find the value of an inverse trigonometric function for an angle shown in the unit circle.

1. $\cos^{-1}\frac{\sqrt{3}}{2}$ = x

domain of cos = 0 < θ < 180°
0 < θ < π

$\theta =$ 30 **in degrees** $\theta =$ $\frac{\pi}{6}$ **in radians**

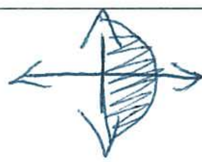
2. $\tan^{-1}\frac{\sqrt{3}}{3}$



→ 30

$\theta =$ 30 **in degrees** $\theta =$ $\frac{\pi}{6}$ **in radians**

3. $\sin\theta = 1$



y-value = 1

$\theta =$ 90 **in radians** $\theta =$ $\frac{\pi}{2}$ **in degrees**

4. $\tan^{-1}\sqrt{3}$

60

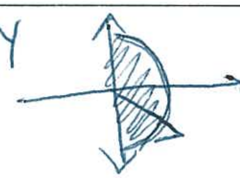
$\theta =$ 60 **in radians** $\theta =$ $\frac{\pi}{3}$ **in degrees**

5. $\cos^{-1}\frac{\sqrt{2}}{2}$ = x

is cos even

$\theta =$ 45 **in radians** $\theta =$ π/4 **in degrees**

6. $\sin\theta = -\frac{1}{2}$ = Y



$\theta =$ 330 **in radians** $\theta =$ $\frac{11\pi}{6}$ **in degrees**

7. $\tan\theta = 1$

$\theta =$ ______ **in radians** $\theta =$ ______ **in degrees**

8. $\cos^{-1}(-\frac{\sqrt{2}}{2})$

$\theta =$ ______ **in radians** $\theta =$ ______ **in degrees**

9. **CHALLENGE EXAMPLE:** Solve for x

$$\sin^{-1}(x-1) = \frac{\pi}{4}$$

➔ (To free the "x-1" I take the **sin** of both sides)

$(x-1) = \sin\frac{\pi}{4}$ (sub in the values I know)

$x = 1 + \frac{\sqrt{2}}{2}$ Easy enough!

10. **CHALLENGE PROBLEM:** You try!

Solve for x in the expression: $\tan^{-1}(x+2) = 1$

QUIZ PRACTICE: No Calculators. SHOW ALL WORK!

Find the **exact** value of each trigonometric function:

15. $\cos \pi$

$= \boxed{-1}$

16. $\sin \frac{7\pi}{4}$

$\boxed{-\frac{\sqrt{2}}{2}}$

17. $\tan \frac{2\pi}{3}$

120°
$=60^\circ$

$= \boxed{-\sqrt{3}}$

18. $\tan \frac{5\pi}{6}$

$= \boxed{-\frac{\sqrt{3}}{3}}$

19. $\sin \frac{10\pi}{8}$

$\sin \frac{5}{4}\pi$

$\boxed{-\frac{\sqrt{2}}{2}}$

20. $\cos \frac{11\pi}{6}$ $=$ y-value

$\boxed{\frac{\sqrt{3}}{2}}$

21. Solve for x:

angle

$\tan^{-1} \tan(x+2) = (1) \leftarrow \tan^{-1}$ $\quad \frac{\pi}{4} - 2$

$x + 2 = \tan^{-1} 1$

$x + 2 = 45$ $\quad x = 43$

Hint: What is the inverse of tan? Use that to "free" x+2

22. **True or False**: $\tan \frac{\pi}{2}$ has an exact value. Explain your answer using the unit circle.

$(0, 1)$

False $\tan \pi/2$ $\quad y/x$

$1/0 =$ undefined

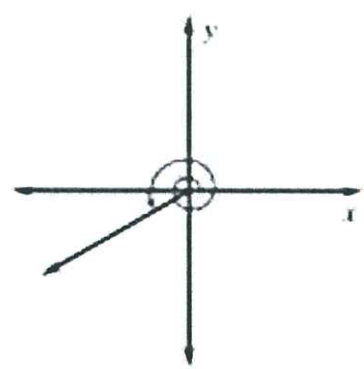
23. For a quadrantal angle, what are the **only** values that can define sine, cosine, and tangent of θ.

1, 0, and undefined

$\boxed{1 \text{ and } 0}$

not a value

24. What is the value of θ in radians for the illustrated angle? Define all 3 trig functions of θ.



25. What is the value of the expression $\sin^{-1}\left(-\frac{\sqrt{2}}{2}\right)$?

26. Solve the equation for θ in degrees:

$$\tan\theta = -\sqrt{3}$$

Name: Key TP: ______

HW#70H: Inverse Trig
Due: Monday, March, 4th
Honors Geometry

Failure to show all work will result in LaSalle. **REMEMBER TO WATCH YOUR SIGNS!**

1) For $0 < x < 2\pi$, if $\sin x < 0$ and $\cos x < 0$, what are the possible values of x? (−) (−)

tan (+)

A. $0 < x < \frac{\pi}{2}$

B. $\frac{\pi}{2} < x < \pi$

C. $\pi < x < \frac{3\pi}{2}$

D. $\frac{3\pi}{2} < x < 2\pi$

E. $0 < x < 2\pi$

S A T C

$\frac{3\pi}{2}$

2) For $0 < x < 2\pi$, if $\tan x > 0$ and $\cos x < 0$, what are the possible values of x?

A. $0 < x < \frac{\pi}{2}$

B. $\frac{\pi}{2} < x < \pi$

C. $\pi < x < \frac{3\pi}{2}$

D. $\frac{3\pi}{2} < x < 2\pi$

E. $0 < x < 2\pi$

S A T C (+)

3) Solve the equation for θ without a calculator. Give your answer in both radians and degrees.

$\sin\theta = (-\frac{\sqrt{3}}{2})$

Domain of sin

−60

$\pi/3$

down here

4) Solve the equation for θ without a calculator. Give your answer in both radians and degrees.

$\tan\theta = (-\sqrt{3})$

−60°

$\pi/3$

5) Solve the equation for θ without a calculator. Give your answer in both radians and degrees.

$\cos\theta = (-\frac{1}{2})$

~~135~~° or $\frac{3\pi}{4}$

6) Solve the equation for θ without a calculator. Give your answer in both radians and degrees.

$\sin\theta = 0$

0°, 0

7) Explain why the domain must be restricted when evaluating inverse trig functions. What restrictions are there on each domain?

It allows the inverse functions to be defined

$\sin^{-1} = -\pi/2 \leq \theta \leq \pi/2$

$\cos^{-1} = 0 \leq \theta \leq 180$

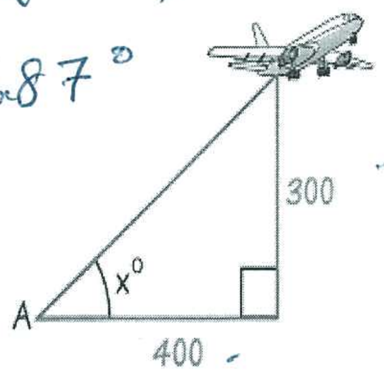
$\tan^{-1} = -\pi/2 \leq \theta \leq \pi/2$

Review Problems:

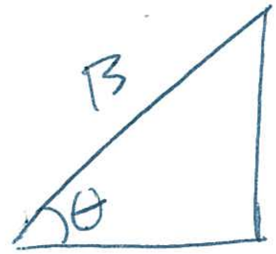
8. Find the size of the angle of elevation of the plane from point A on the ground.

$\tan^{-1} = \left(\frac{300}{400}\right)$

$= 36.87°$



9. If in a right triangle $\tan\theta = \frac{x}{5}$ and $\sin\theta = \frac{x}{13}$, then x = ?



$x = 12$

$25 + x^2 = 169 \quad x^2 =$

Draw the triangle!

$x = 12$

10. Find the **exact** value of tan $\frac{\pi}{4}$ + sin 30°.

$\tan \pi/4 - \sin 30$

$45°$

$1 - 1/2 = 1/2$

__Prove__ your answer using special right triangles.

11. Convert each angle in degrees to radians. Express your answer in simplest form __and__ in terms of π.

a) 17°

$17\pi/180$

b) -40°

$-2\pi/9$

c) 125°

$25\pi/36$

d) 270°

$3\pi/2$

12. Convert each radian to degrees. Round your answer to the nearest degree.

a) 3.14

π

b) 0.75

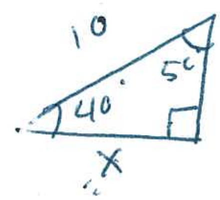
$42.3°$

c) 6.32

$362°$

d) $\sqrt{2}$

13. A right triangle has a hypotenuse of length 10 centimeters. If one angle is 40°, find the length of each leg.



y

$\sin 50 = x/10$

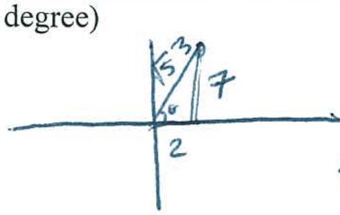
$x = 7.66$

$y = 6.43$

$\sin 40 = y/10$

14. The point (x,y) is on the terminal side of an angle θ in standard position. Find the exact value of the given trigonometric function **and** θ. (Round θ to the nearest tenth degree)

(2,7), Sin θ



$4 + 49 = \sqrt{53}$

$\sin = 7/\sqrt{53} = \frac{7\sqrt{53}}{53}$